## Muon capture on neutron-rich nuclei

E. Kolbe<sup>1</sup>, K. Langanke<sup>2,a</sup>, and K. Riisager<sup>2</sup>

<sup>1</sup> Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831-6373, USA

<sup>2</sup> Institute for Physics and Astronomy, University of Aarhus, DK-8000 Aarhus, Denmark

Received: 6 April 2001 / Revised version: 8 May 2001 Communicated by J. Äystö

**Abstract.** Future facilities will allow the exploration of extremely neutron-rich nuclei far from the valley of stability. It is discussed that the strong-neutron excess results in changes in the collective excitations of such nuclei compared to conventional stable nuclei. We propose muon capture as an experimental tool to explore such changes. We will quantify our discussion by the calculation of the total and differential muon capture rates on selected calcium isotopes between <sup>40</sup>Ca and <sup>60</sup>Ca. Our calculations are based on the random phase approximation and agree nicely with the measured rates for <sup>40</sup>Ca and <sup>44</sup>Ca.

**PACS.** 24.30.Cz Giant resonances – 23.40.-s Beta decay; double beta decay; electron and muon capture – 23.40.Hc Relation with nuclear matrix elements and nuclear structure

A major long-term challenge for future nuclear research is the exploration of the extremely neutron-rich region of the nuclear chart [1]. Decisive advances into this terra incognita are expected from novel experimental facilities which are either under construction like the MSU/NSCL and Riken upgrades or in the proposal phase like the GSI upgrade, the Rare Isotope Accelerator, or the Eurisol project. One of the goals at these facilities will be to search for new modes of collective motion which can be quite different from what is seen in normal nuclei [2]. This expectation is based on the consideration that in nuclei with large neutron excess, the proton and neutron liquids can have quite different properties. It is predicted, for example, that a vibration of the neutron skin against the core can shift strength from the normal giant-dipole region to much lower excitation energies, giving rise to what is called a pygmy resonance [3–5]. Furthermore, models find the giant-dipole resonance much stronger fragmented than in normal nuclei. A similar shift of the collective strength to lower energies is also predicted for other collective excitation modes, like the isoscalar monopole strength [2].

New collective modes or changes in the collective strength response have important astrophysical implications. So can the existence of pygmy resonances drastically increase the dipole strength around the neutron threshold, which, as a consequence, significantly increases the neutron capture cross-section for the neutron-rich nuclei along the r-process path [6]. The collective dipole and spin-dipole response dominates the neutral-current crosssection for neutrino reactions on nuclei induced by supernova  $\nu_{\mu}$  and  $\nu_{\tau}$  neutrinos [7]. As supernova  $\nu_{e}$  neutrinos are expected to have lower energies, the  $(\nu_{e}, e^{-})$  chargedcurrent reactions are dominated by allowed collective transitions [8]. In the exciting case of neutrino oscillations [9, 10], however, also the charged-current cross-sections can be strongly influenced by dipole transitions, with interesting consequences and constrains for both neutrino properties and supernova models if the nuclear *r*-process occurs in the layer above the newly born neutron star in a corecollapse supernova [11]. The giant-monopole resonance is traditionally used to determine the compressibility of nuclear matter [12]. Finally we mention that collective modes usually also dominate the electron capture and  $\beta$  decay cross-sections during the core collapse of a massive star [13,14].

Next-generation experimental facilities plan to study the collective modes in neutron-rich nuclei in inverse kinematics in which the projectile has to be excited. As possible tools, charge-exchange reactions, inelastic electron scattering, or Coulomb excitation has extensively been discussed. In this paper, we propose muon capture as an alternative method to study collective modes in these nuclei.

The capture of a negative muon from the atomic 1s orbit

$$\mu^{-} + (Z, N) \to \nu_{\mu} + (Z - 1, N + 1)^{*}$$
 (1)

is a semileptonic weak process which has been studied for a long time (see, *e.g.*, the reviews by Walecka [15], Mukhopadhyay [16], or Measday [17] and the earlier references therein). Thus, the energy ( $\omega$ ) and momentum (q) transfer is of the order of the muon mass (~ 105 MeV).

<sup>&</sup>lt;sup>a</sup> e-mail: langanke@ifa.au.dk

This makes the process favorable for the study of forbidden transitions which are enhanced by phase space compared with allowed transitions. In this paper, we use the calcium isotopes to demonstrate the potential of muon capture to explore forbidden responses in neutron-rich nuclei. For both <sup>40</sup>Ca and <sup>48</sup>Ca it has been shown before that muon capture is dominated by dipole and spin-dipole responses [18]. However, we will argue that the relative contribution of the various forbidden transitions vary when systematically studying muon capture within a chain of isotopes.

Our calculations are based on the random phase approximation, which despite its simplicity, has been shown previously to yield quite satisfactory descriptions for muon capture on nuclei, e.q. [20,18]. We have adopted the same approach as described in [19,20,18], including the partial occupancy formalism. It has been shown in [18] that the discrete and continuum models of the RPA give very similar results for muon capture rates and excitation spectra. We thus have used the numerically much simpler discrete version in this manuscript. We have considered nuclear transitions mediated by the q-dependent multipole operators of both parity and  $J \leq 4$  as defined in [15]. The single-particle energies are derived from the parametrization of [21]. As a residual interaction, we used a zero-range Migdal force [22]. It has been shown that calculations with a finite-range interaction derived from the Bonn potential [22] give very similar results for the muon capture on  $^{40}$ Ca [20]. The muon wave function in the atomic (1s) orbital has been derived by solving the Dirac equation for an extended charge distribution. Spurious center-of-mass motion can contaminate the  $1^-$  RPA excitation spectra. This contamination is particularly important in the isoscalar channel, while muon capture is dominated by isovector excitations. Furthermore, we have removed from our calculations the eigenstates which correspond to the spurious center-of-mass motion.

Muon capture also depends on the induced pseudoscalar hadronic weak current. At the free nucleon level the corresponding coupling constant is determined by the Goldberger-Treiman relation [23]

$$F_P(q^2) = \frac{2M_p F_A(0)}{m_\pi^2 - q^2} , \qquad (2)$$

where  $m_{\pi}$  is the pion mass and  $F_A(0) \equiv g_A = 1.25$ . (In muon capture one often uses a dimensionless quantity  $g_P = m_{\mu}F_P(q^2)$  at the relevant momentum transfer  $q^2 \simeq -0.9m_{\mu}^2$ , such that  $g_P \simeq 8.4$  for free protons.) In nuclear medium  $F_P$  can be again renormalized, and this renormalization does not necessarily obey the Goldberger-Treiman relation [24]. We have shown in our previous work that the total muon capture rates are not sensitive enough to the various choices of  $F_P$  renormalization. Consequently, throughout this work we use the Goldberger-Treiman relation.

Hamamoto and collaborators [2] have studied the isovector and isoscalar dipole response in <sup>60</sup>Ca performing Hartree-Fock calculations with Skyrme interactions and then using the random phase approximation. A



Fig. 1. Excitation spectrum of the  $^{60}$ Ca nucleus for photo absorption. The results have been folded with a Gaussian of width 0.5 MeV.

Table 1. Calculated total muon capture rates. The rates are given in  $10^5 \text{ s}^{-1}$ .

Target nucleus	Calculated rate
$^{40}$ Ca	25.5
$^{44}Ca$	17.2
$^{48}Ca$	13.1
$^{52}Ca$	8.67
$^{56}$ Ca	5.79
<sup>60</sup> Ca	2.29

comparison to this work allows us to test our singleparticle parametrization. Figure 1 shows our calculated <sup>60</sup>Ca photo absorption cross-section which exhibits two peaks between E = 7.5-11 MeV and 14–18 MeV, respectively. These structures are in good agreement with the maxima of the isoscalar dipole response (around 8 MeV) and the isovector dipole response (around 15 MeV) as predicted in [2].

Our total muon capture rates  $\Lambda$  for the selected calcium isotopes are summarized in table 1. We note that the calculated rates agree very well with the experimental data [25] for <sup>40</sup>Ca ( $\Lambda = (25.44 \pm 0.07) \times 10^5 \text{ s}^{-1}$ , corrected from the data for natural calcium) and <sup>44</sup>Ca ((17.93 \pm 0.40) × 10<sup>5</sup> s<sup>-1</sup>).

It has long been recognized [26] that total muon capture rates on a nucleus with charge and mass numbers Zand A are well described by the rather simple Primakoff rule

$$\Lambda(A,Z) = Z_{\text{eff}}^4 X_1 \left( 1 - X_2 \frac{(A-Z)}{2A} \right).$$
 (3)

A pedagogical derivation of this formula is given in [17]. We note that the second term in the parentheses corrects for Pauli blocking in the final nucleus. The use of an effective charge number  $Z_{\rm eff}$  rather than Z accounts for corrections needed as the nuclear and muonic radii are comparable. In fig. 2 we compare our total muon capture rates with



Fig. 2. Total muon capture rates (circles) and the  $J = 1^{-1}$  (squares) and  $2^{-1}$  (diamonds) multipole contributions for selected calcium isotopes as a function of (A - Z)/A.

the Primakoff prediction and find that the proportionality  $\Lambda \sim (A-Z)/A$  holds rather well.

However, the slope is quite different from the one seen in normal nuclei. The calculations for <sup>40</sup>Ca and <sup>44</sup>Ca fit nicely with the trend seen for other beta-stable nuclei (that reach out to (A - Z)/A = 0.61), where parameter values are [25]  $X_1 = 170 \text{ s}^{-1}$  and  $X_2 = 3.13$ . In contrast, our calculations give  $X_1 = 124 \text{ s}^{-1}$  and  $X_2 = 2.91$  for the isotopes <sup>44</sup>Ca to <sup>60</sup>Ca. Extrapolation of the formula fitted to beta-stable nuclei would give a negative capture rate already at <sup>56</sup>Ca; such a simple extrapolation of systematics gathered for beta-stable nuclei breaks down for the neutron-rich nuclei. In spite of the Pauli blocking we find that even for <sup>60</sup>Ca, with twice as many neutrons as protons, the muon capture rate is still comparable with the natural muon decay rate  $4.5516 \times 10^5 \text{ s}^{-1}$ . Nuclear muon capture thus remains very probable even for very neutron-rich nuclei.

Despite the simple Primakoff scaling rule, an analysis reveals that the total muon capture rates depend on detailed nuclear structure effects. This is demonstrated in fig. 2 by plotting the partial  $J = 1^-$  and  $2^-$  contributions to the total capture rate. We clearly observe that the  $1^-$  contribution scales nicely following the Primakoff rule, while the  $2^-$  contribution clearly does not obey this scaling rule. To make this even more transparent, fig. 3 shows the relative contributions of the various multipole transitions to the rate. Before discussing the results, we note that our chain of calcium isotopes includes two nuclei,  $^{40}$ Ca and  $^{60}$ Ca, with doubly closed shell structures. As a consequence, allowed Gamow-Teller–like excitations vanish in the independent particle model. Furthermore, the closed neutron pf shell suppresses  $1\hbar\omega$  excitations in  $^{60}$ Ca.

Upon inspection of figs. 2 and 3, we find that more than 70% of the total capture rate in <sup>40</sup>Ca is due to  $J^{\pi} = 1^{-}$  and  $2^{-}$  multipole contributions. (These multipoles also dominate the rate for nuclei like <sup>12</sup>C, where Gamow-Teller contributions are allowed.) The  $1^{-}$  multipole also yields the largest portion to the capture rates in the isotopes



Fig. 3. Fractional contribution of different multipoles to the total capture rate for the six selected calcium isotopes. The entries with  $J^{\pi} = 0^{-}, 1^{-}, 2^{-}$  correspond to the  $1\hbar\omega$  excitations, while the entries with  $J^{\pi} = 0^{+}, 1^{+}, 2^{+}, 3^{+}$  correspond to the  $2\hbar\omega$  excitations.

 $^{44,48,52,56}\mathrm{Ca.}$  As it mainly involves collective  $1\hbar\omega$  excitations, it is suppressed in <sup>60</sup>Ca. Upon inspection of our RPA results, we find that the  $1^-$  multipole contribution is dominated by collective transitions. The  $2^-$  multipole contributes significantly to <sup>40</sup>Ca, but its relative importance reduces noticeably with increasing neutron excess. The reason is that this response has very strong singleparticle transitions at rather low excitation energies, involving in particular  $d_{5/2} \rightarrow f_{7/2}$  and  $d_{5/2} \rightarrow p_{3/2}$  transitions. These transitions are favored as the initial and final radial single-particle wave functions  $u_i, u_f$  have no nodes and hence  $\int u_i r u_f dr$  is large. However, the final states become increasingly Pauli-blocked with increasing neutron excess which reduces the fractional weight of these single-particle transitions, and hence of the  $2^-$  multipole, to the total capture rate. As mentioned above, the GTlike transitions are strongly suppressed in muon capture on calcium. Hence, the  $1^+$  contributions reflect mainly operators like  $rp\sigma$  and  $r^2\sigma$ . The latter is the multipole dominating the collective response in <sup>60</sup>Ca within our calculation, followed by the  $2\hbar\omega J^{\pi} = 0^+$  multipole.

We summarize that the  $1^-$  multipole mainly reflects collective contributions, while the  $2^-$  is dominated by single-particle transitions. The fact that the position of the collective  $1^-$  modes in the isotope chain (as measured from the parent ground state) varies only slightly and regularly and the total strength obeys a sum rule explains why this multipole follows the Primakoff scaling rule. However, the  $2^-$  response is quite sensitive to nuclear structure as it involves single-particle transitions. (Incidentally, our results for the low-lying transitions agree qualitatively with the detailed experimental results for  ${}^{40}$ Ca [17,27].) This dependence spoils the Primakoff scaling for this multipole. Our predictions might already be checked by measuring the muon capture rate on  ${}^{48}$ Ca which is predicted to be larger than the one extrapolated from the  ${}^{40}$ Ca- ${}^{44}$ Ca data (and calculation) by the Primakoff rule.



Fig. 4. Excitation spectrum for contributions of the  $J^{\pi} = 1^+$  multipole operator to the muon capture rate for selected calcium isotopes (upper part: <sup>40</sup>Ca (left), <sup>44</sup>Ca (right); middle part: <sup>48</sup>Ca (left), <sup>52</sup>Ca (right); lower part: <sup>56</sup>Ca (left), <sup>60</sup>Ca (right)). The RPA results have been folded with a Gaussian distribution of width 1 MeV.

The excitation spectra for the most important multipoles,  $J = 1^+, 1^-$ , and  $2^-$ , are shown in figs. 4-6. We note that for all these multipoles the response moves to significantly smaller excitation energies with increasing neutron excess from <sup>40</sup>Ca to <sup>56</sup>Ca. The trend is then broken for <sup>60</sup>Ca, due to the neutron pf shell closure in this nucleus. The neutron separation energy in the final state K isotopes also decreases (they are estimated as 2.3 MeV [28], 1.68 MeV, and 0.16 MeV [29] for A = 52, 56, 60, respectively), but not as rapidly as the response, so one would expect the average number of emitted neutrons to decrease as we go to more neutron-rich isotopes and then a sudden increase for <sup>60</sup>Ca.

We note again that our simple RPA calculation predicts a neutron shell closure at N = 40. However, recent data indicate that the N = 20 and N = 28 neutron shell closures get eroded in very neutron-rich nuclei [30,31]. This is explained by strong proton-neutron crossshell correlations which pull protons into the higher shell [32–34]. If a similar effect occurs in the very neutronrich calcium isotopes, *i.e.* protons being pulled into the pfshell, Gamow-Teller transitions might contribute to the capture on <sup>52,56</sup>Ca and first-forbidden transitions will not be suppressed in <sup>60</sup>Ca. As a consequence, the capture rates for these nuclei might go up, which can be reflected by deviations from the simple Primakoff rule. Furthermore, a sizable dipole strength in <sup>60</sup>Ca should show up at rather low excitation energies associated with a low neutron multiplicity in the decay.

Another topic of considerable interest in nuclear structure is the possible renormalization of the axial vector

coupling constant  $g_A$  in the nuclear medium. This effect is well established for Gamow-Teller transitions [35,36], and evidence for quenching has also been reported recently for M2 transitions [37]. The quenching seems to be absent for first-forbidden transitions [20, 18], and the situation for higher multipoles is still unclear. As the multipole decomposition of the muon capture rate within an isotope chain strongly varies, quenching of certain multipole transitions might lead to deviations from the Primakoff rule. It might be possible to investigate this effect also for the low-lying transitions which, for most multipoles, have single-particle excitation character and therefore a relatively simple nuclear structure. As the neutron thresholds for the very neutron-rich nuclei will be quite low, muon capture will lead in most cases to final nuclear states which decay by neutron emission. Hence, observations of these decays can allow us to determine individual transition rates.

If individual (excited) final states can be observed, we gain structural information in a nucleus "one step more neutron-rich". The other standard tools for obtaining similar information, such as (n, p) or  $(d, {}^{2}\text{He})$  reactions [17], would have to be performed in inverse kinematics and would be quite challenging. We, therefore, turn now to a brief consideration of the experimental feasibility of the muon capture reactions.

Experiments would have to work with limited amounts of muons and radioactive ions. One possibility is to merge muons and radioactive ions at low relative velocity in traps or in storage rings. The alternative possibility of muon transfer inside a solid hydrogen film is discussed in [38]. For sufficiently low relative energy, muon capture on an



Fig. 5. Same as fig. 4, except for the  $J^{\pi} = 1^{-}$  multipole operator.



Fig. 6. Same as fig. 4, except for the  $J^{\pi} = 2^{-}$  multipole operator.

ion would have a cross-section of the order of the ion area, and atomic muon capture will take place. Rough estimates [39] indicate that future facilities should be able to reach a production rate for muonic atoms larger than one per second, but also indicate that only a few of the ions and a tiny fraction of the muons will participate in atomic capture reactions. As mentioned above, the formed muonic atoms will, in most cases, lead to nuclear muon capture.

There will be a substantial experimental background from decays of the non-captured muons and from the other ions. Physical separation of the muonic atoms from the non-captured muons would help greatly, but it seems prudent in any case to include detection, both of the X-rays from the muonic atom and the nuclear decay products from the nuclear muon capture that takes place up to a few microseconds later. For Ca the X-rays will have energies from 784 keV to about 1 MeV [17]. A measurement of the effective muon half-life via the electron decay channel (a technique often employed for stable nuclei) is not likely to give a measureable signal due to the small proportion of muonic atoms formed. The nuclear decay products should thus be measured efficiently. This should not present problems, since detailed beta decay spectroscopy nowadays is possible both for proton-rich and neutronrich isotopes produced with a few atoms per second; see, e.g., [40,41]. Detection techniques to be used in the trap environment are also being developed.

Since the muon neutrino even for  $^{60}$ Ca is predicted to take away most of the available energy, we expect the recoil momentum of the nucleus, after the capture, will be of the order of 50 MeV/c or more. A similar momentum will be given to the nucleus when emitting a 1–2 MeV neutron. For the mass range we discuss, the nucleus will thus get a recoil energy of the order of several tens of keV. It can therefore escape an ion trap and this could be used as a further signal for the capture process. Measurements of the isotopic distribution after capture could also be envisaged but would clearly be difficult.

Going all the way to the neutron drip line, we could meet halo structures. There has actually already been muon capture experiments performed on <sup>11</sup>B in which the two bound halo states in <sup>11</sup>Be are reached. A decreased capture rate due to the halo has been predicted, but the experiments are not yet unequivocal [17]. Capture reactions on the radioactive nuclei  $^{6,8}$ He and  $^{11}$ Be, going to the halo nucleus  $^{11}$ Li or on  $^{11}$ Li itself, could be very interesting. Similar effects could be expected to take place in heavier, very neutron-rich, nuclei, but the potential strength in muon capture could turn out to lie more in the transitions to higher excited states that are hard to reach in a clean way otherwise. The physics of the barely unbound states (and nuclei) is closely related to the physics in the barely bound nuclei (halos as well and others) and is already being investigated closely for the very light nuclei. Nuclear reactions involving drip line nuclei have so far mainly given information on the structure in the original nucleus at excitation energies up to about 5 MeV, higher excitation energies can be reached in the produced nuclei or through beta decays. Muon capture will reach

more neutron-rich isotopes and allow us to see structural features at excitation energies up to several tens of MeV.

In conclusion, we have shown that muon capture should proceed with a measurable rate even for very neutron-rich nuclei, and that one would expect nuclear structure changes to be reflected in the capture rates more clearly in the differential rates, but also to some extent in the total rates. Experiments to determine the capture rates will be hard but could become feasible at nextgeneration radioactive beam facilities. The isospin dependence of the total capture rate has only been tested experimentally for a limited range of isotopes [16]. We have argued that the dependence in the Ca chain is likely to differ from the one seen in beta-stable nuclei; it would be interesting to explore this question in more detail also for other isotopic chains.

We would like to thank Jules Deutsch for discussions and Jacek Dobaczewski, Witek Nazarewicz and Petr Vogel for useful comments on our manuscript. This work was supported in part by the Danish Research Council, and in part by the Division of Nuclear Physics, U.S. Department of Energy under contract No. DE-AC05-00OR22725 managed by UT-Battelle, LLC.

## References

- 1. Scientific Opportunities with Fast Fragmentation Beams from the Rare Isotope Accelerator, National Superconducting Cyclotron Laboratory, Michigan State University, 2000.
- H. Sagawa, I. Hamamoto, X.Z. Zhang, J. Phys. G 24, 1445 (1998).
- P. van Isacker, M.A. Nagarajan, D.D. Warner, Phys. Rev. C 45, R13 (1992).
- Y. Suzuki, K. Ikeda, H. Sato, Progr. Theor. Phys. 83, 180 (1990).
- F. Catara, E.G. Lanzo, M.A. Nagarajan, A. Vitturi, Nucl. Phys. A 624, 449 (1997).
- 6. S. Goriely, Phys. Lett. B 436, 10 (1998).
- S.E. Woosley, D.H. Hartmann, R.D. Hofmann, W.C. Haxton, Astrophys. J. 356, 272 (1990).
- A. Hektor, E. Kolbe, K. Langanke, J. Toivanen, Phys. Rev. C 61, 055803 (2000).
- B.S. Meyer, G.C. McLaughlin, G.M. Fuller, Phys. Rev. C 58, 3696 (1998).
- G.C. McLaughlin, J. Fetter, B. Balantekin, G.M. Fuller, Phys. Rev. C 59, 2873 (1999).
- S.E. Woosley, J.R. Wilson, G.J. Mathews, R.D. Hofman, B.S. Meyer, Astrophys. J. **433**, 229 (1994).
- D.H. Youngblood, H.L. Clark, Y.W. Lui, Phys. Rev. Lett. 82, 691 (1999).
- G.M. Fuller, W.A. Fowler, M.J. Newman, Astrophys. J. 293, 1 (1985) and references therein.
- K. Langanke, G. Martinez-Pinedo, Nucl. Phys. A 673, 481 (2000).
- J.D. Walecka in *Muon Physics II*, edited by V.W. Hughes, C.S. Wu (Academic Press, New York, 1975) p. 113.
- 16. N.C. Mukhopadhyay, Phys. Rep. C 30, 1 (1977).
- 17. D.F. Measday, The nuclear physics of muon capture (2001), work in preparation.

- E. Kolbe, K. Langanke, P. Vogel, Phys. Rev. C 62, 055502 (2000).
- E. Kolbe, K. Langanke, S. Krewald, F.K. Thielemann, Nucl. Phys. A 540, 599 (1992).
- E. Kolbe, K. Langanke, P. Vogel, Phys. Rev. C 50, 2576 (1994).
- 21. J. Duflo, A.P. Zuker, Phys. Rev. C 59, R2347 (1999).
- 22. K. Nakayama, S. Drozdz, S. Krewald, J. Speth, Nucl. Phys. A 470, 573 (1987).
- M.L. Goldberger, S.B. Treiman, Phys. Rev. 111, 354 (1958).
- 24. J. Delorme, M. Ericson, Phys. Rev. C 49, R1763 (1994).
- T. Suzuki, D.F. Measday, J.P. Roalsvig, Phys. Rev. C 35, 2212 (1987).
- 26. H. Primakoff, Rev. Mod. Phys. 31, 802 (1959).
- 27. P. Igo-Kemenes, J.P. Deutsch, D. Favart, L. Grenacs, P. Lipnik, P.D. Macq, Phys. Lett. B 34, 286 (1971).
- G. Audi, O. Berrillon, J. Blachot, A.H. Wapstra, Nucl. Phys. A 624, 1 (1997).

- P. Möller, J.R. Nix, K.L. Kratz, At. Data Nucl. Data Tables 66, 131 (1997).
- 30. T. Motobayashi et al., Phys. Lett. B 346, 9 (1995).
- 31. H. Scheit et al., Phys. Rev. Lett. 77, 3967 (1996).
- 32. A. Poves, J. Retamosa, Nucl. Phys. A 571, 221 (1994).
- N. Fukunishi, T. Otsuka, T. Sebe, Phys. Lett. B 296, 279 (1992).
- D.J. Dean, M.T. Ressell, M. Hjorth-Jensen, S.E. Koonin, K. Langanke, A.P. Zuker, Phys. Rev. C 59, 2474 (1999).
- 35. B.H. Wildenthal, Progr. Part. Nucl. Phys.  $\mathbf{11},\,5$  (1984).
- 36. K. Langanke, D.J. Dean, P.B. Radha, Y. Alhassid, S.E. Koonin, Phys. Rev C 52, 718 (1995).
- P. von Neumann-Cosel et al., Phys. Rev. Lett. 82, 1105 (1999).
- P. Strasser, T. Matsuzaki, K. Nagamine, Hyperfine Interact. 119, 317 (1999).
- 39. K. Jungmann, unpublished.
- 40. H.O.U. Fynbo et al., Nucl. Phys. A 677, 38 (2000).
- 41. U.C. Bergmann et al., Nucl. Phys. A 658, 129 (1999).